Model Question Paper

Reg No:

Name:

RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY

(AUTONOMOUS)

FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2021

100908 /MA100A LINEAR ALGEBRA & CALCULUS

Max. Marks: 100 **Duration: 3 hours**

PART A

(Answer **all** questions, **each** question carries 3 marks)

1. Determine the rank of the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.
2. Write down the Eigen values of $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.
3. Find $f_x(1,3)$ and $f_y(1,3)$for the function $f(x,y) = 2x^3y^2 + 2y + 4x + 3$.
4. If $w = x^2 + y^2 - z^2$ where$x = \rho \sin\emptyset \cos\theta, y = \rho \sin\emptyset \sin\theta\,, z = \cos\phi$. Use appropriate form of the chain rule to find $\frac{\partial w}{\partial \rho}, \frac{\partial w}{\partial \theta}$ and $\frac{\partial w}{\partial \emptyset}$.
5. Use double integral to find the area of the region enclosed between the parabolas $y = x^2$ and the line $y = 2x$.
6. Use polar coordinates to evaluate the area of the region bounded by$x^2 + y^2 = 4$, the line $y = x$ and the y axis in the first quadrant.
7. Test the convergence of the series$\sum_{K=1} \frac{k}{k+1}$.
8. Test the convergence of the alternating series$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$ using Leibnitz test.
9. Find the Taylor series expansion of $sin\pi x$ about $x = \frac{1}{2}$.
10. Find the values to which the Fourier series of $f(x) = x$ for $-\pi < x < \pi$, with $f(x + 2\pi) = f(x)$ converges.

PART B

(Answer **one full** question from each module, each question carries **14** marks)

Module -I

11. (a) Solve the following system of equations

$$y + z - 2w = 0$$

$$2x - 3y - 3z + 6w = 2$$

$$4x + y + z - 2w = 4$$

(b) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

12. (a) Diagonalize the matrix $A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

(b) What kind of conic section the quadratic form $3{x_1}^2 + 22x_1x_2 + 3{x_2}^2$ =represents?

Transform it to Principal axes.

Module – II

13. (a) Find the local linear approximation to $f(x,y) = x^2 + y^2$ at the point $(3,4)$.Use it to approximate $\overline{(3.04,3.98)}$.

(b) Let $w = x^2 + y^2 + z^2, x = cos\theta, y = sin\theta,\ z = tan\theta$. Use chain rule to find $\frac{dw}{d\theta}$ when $\theta = \frac{\pi}{4}$

14. (a) Let $z = f(x,y)$ where $x = rcos\theta, y = rsin\theta$, prove that,

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

(b) Locate all relative maxima, relative minima and saddle points $f(x,y) = xy + \frac{a^2}{x} + \frac{b^2}{y}, a \neq 0, b \neq 0.$

Module – III

15. (a) Evaluate $\iint (2x^2y + 9y^3)\, dxdy$ where D is the region bounded by $y = 2x$ and $y = 2\sqrt{x}$

(b) Evaluate $\int_0^4 \int_{\sqrt{y}}^{2} e^{x^2}\, dxdy$ changing the order of integration.

16. (a) Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(b) Evaluate $\iiint (1 - x^2 - y^2 - z^2)\, dxdydz$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$, by transforming to spherical polar coordinates

Module - IV

17. (a) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k^k}{k!}$

(b) Determine the convergence or divergence of the series $\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$

18. (a) Check whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(2k)!}{(3k-2)!}$ is absolutely convergent, conditionally, convergent or divergent.

(b) Test the convergence of the series $1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \cdots$..

Module - V

19. (a) Obtain the Fourier series of for $f(x) = e^{-s}$, in the interval $0 < x < 2\pi$ with

$f(x + 2\pi) = f(x)$ Hence deduce the value of $\sum_{n=2}^{\infty} \frac{(-1)^n}{1+n^2}$

(b) Find the half range sine series of $f(x) = 2x \; in \; (0, l)$

20. (a) Expand $(1 + x)^{-2}$.as a Taylor series about $x = 0$ and state the region of convergence of the series.

(b) Find the Fourier series for $f(x) = x^2$ in the interval $-\pi \leq x < \pi$ with

$(x + 2\pi) = f(x)$.Hence show that $\frac{1}{1^4} + \frac{1}{2^4} + \cdots = \frac{\pi^4}{90}$.